Written Exam for the M.Sc. in Economics, Winter 2010/2011

# Contract Theory

Final Resit Exam / Master's Course

February 17, 2011

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

**Attempt both questions**

**Question 1**

There are two countries, $A$ (which is rich) and $B$ (which is poor). A monopoly firm can produce and sell its good — a particular computer program — in the two countries. The program can be offered in different qualities. The payoff of a representative consumer in country $i \in \{A, B\}$, if consuming the program at price $p$ when the quality is $q$, is given by

$$U_i = \theta_i q - p,$$

where it is assumed that

$$\theta_A > \theta_B > 0.$$

The program can be offered at different prices and different qualities in the two countries. However, a consumer can, without incurring any transaction costs, choose to purchase the program in the country in which he is not living, if he prefers that price-quality combination. Therefore, if the firm offers the good in both countries, the following two incentive compatibility constraints must be satisfied:

$$\theta_A q_A - p_A \geq \theta_A q_B - p_B \tag{IC-A}$$

and

$$\theta_B q_B - p_B \geq \theta_B q_A - p_A. \tag{IC-B}$$

In addition, if the firm wants the consumers in both countries to actually purchase the program, their individual rationality constraints must be satisfied. Assuming that all consumers' outside option yields the payoff zero, these constraints can be written as

$$\theta_A q_A - p_A \geq 0 \tag{IR-A}$$

and

$$\theta_B q_B - p_B \geq 0. \tag{IR-B}$$

In addition, in country $B$ there is, for any given quality of the program, a maximum price that the firm is allowed to charge. The level of the price cap is linear in the quality and it can be written as

$$p_B \leq d_B q_B, \tag{PC-B}$$

where

$$0 < d_B < \theta_B.$$

The monopoly firm has a quadratic cost function and wants to maximize its profits. These profits can be written as

$$V = \nu \left( p_A - \frac{c}{2} q_A^2 \right) + (1 - \nu) \left( p_B - \frac{c}{2} q_B^2 \right)$$

where $c > 0$ is a parameter, $\nu \in (0, 1)$ is the number of consumers in country $A$, and $(1 - \nu)$ is the number of consumers in country $B$.

a) Let the first-best levels of $q_A$ and $q_B$ be defined as the ones that maximize the total surplus,

$$(1-\nu)\,\theta_B q_B + \nu\theta_A q_A - (1-\nu)\,\frac{c}{2}q_B^2 - \nu\frac{c}{2}q_A^2.$$

Calculate these first-best levels. Explain the economic intuition behind your result.

b) Return to the model with asymmetric information described above and solve for the optimal second-best prices and qualities. Assume that the parameters of the model are such that the monopoly firm optimally sells in both countries. Explain how the optimal second-best qualities differ from the optimal first-best qualities. Also explain the economic intuition behind any differences. Do the consumers of any one of the countries get any rents at the second-best optimum? If so, which country or which countries? Explain why.

**Question 2**

Prometheus Sørensen (the principal, $P$ for short) owns a factory producing pencils and wants to hire Absalon Nielsen (the agent, $A$ for short) to work there. If hired, $A$'s task will be to operate a pencil machine and to make sure it runs smoothly. To do this well, $A$ must "make an effort", which involves a (personal) cost to $A$. This is modelled as $A$'s choosing an effort level $e \in [0,1]$. The associated cost equals $\psi(e)$, where this function satisfies

$$\psi' > 0, \quad \psi'' > 0, \quad \psi(0) = \psi'(0) = 0, \quad \lim_{e\to 1} \psi'(e) = \infty.$$

The number of pencils that come out of the machine, $q$, is either large ($q = \overline{q}$) or small ($q = \underline{q}$), with $\overline{q} > \underline{q} > 0$. The probability that the number is large equals the effort level: $\Pr(q = \overline{q} \mid e) = e$. $P$ (and the court) can observe which quantity that is realized ($\overline{q}$ or $\underline{q}$) but not the effort level chosen by $A$. It is assumed that $P$ has all the bargaining power and makes a take-it-or-leave-it offer to $A$. A contract can specify two numbers, $\overline{t}$ and $\underline{t}$, where $\overline{t}$ is the payment to $P$ if $q = \overline{q}$, and $\underline{t}$ is the payment to $A$ if $q = \underline{q}$. $P$ is risk neutral and his payoff, given a quantity $q$ and a payment $t$, equals

$$V = q - t.$$

$A$ is also risk neutral and his payoff, given a payment $t$ and an effort level $e$, equals

$$U = t - \psi(e).$$

$A$ is protected by limited liability, meaning that $\overline{t} \geq 0$ and $\underline{t} \geq 0$. A's outside option would yield the payoff zero.

a) Let the first best effort level be defined as the one that maximizes the expected total surplus,

$$(1-e)\,\underline{q} + e\overline{q} - \psi(e).$$

Characterize this first best level. Explain the economic intuition behind your result.

b) Derive the second-best solution; that is, characterize the optimal menu of contracts under the assumption that $P$ (and the court) cannot observe the effort level $e$. Explain the economic intuition behind your result.

END OF EXAM